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Integration Model of Dynamic Inventory Replenishment and Pricing Based on Estimating Demand Substitution for PC Products

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ABSTRACT

Motivated by the recent supply chain management practice of the Chinese PC industry, we examine the impact of demand forecasting and demand substitution estimation on inventory management and pricing with short selling seasons for a PC supply chain consisting of one retailer and two manufacturers. Based on PC products' demand characteristics, product life cycle (PLC) is incorporated into the traditional multiplication demand model and a multi-period dynamic inventory and pricing integrated decision model is developed. This model assumes that retailers always place an initial order at pre-season, and the order quantity would be split into multiple batches following the retailers' dynamic inventory replenishment requirements under the constraint that the sum of the split orders should be almost the same as the initial order. Meanwhile, the retailer would dynamically determine the selling price according to effective demand and demand forecasting per cycle, and the manufacturers decide the wholesale price and rebate depending upon the retailer's total order quantities. Finally, empirical analysis is given, and analysis results show that the model enjoys good feasibility and effectiveness, that can overcome the negative impact of high demand fluctuations on profits and service levels.

Keywords: Dynamic Inventory Decision, Dynamic Pricing, Demand Forecasting, Product Life Cycle, Demand Substitution

INTRODUCTION

The perfect balance of supply and demand is a very difficult goal to achieve in a PC product supply chain because of short product life cycle, high technical upgrade, and diversification of demand, which limits the quick demand response and even risks loss of profit. Stimulated by the recent supply chain management practice of the Chinese PC industry, more and more PC enterprises are thinking of starting to transfer their focus from supply chain management to demand chain management, such as how Dell proposed consumer-centric supply chain management, demand-leading supply chain innovation management to provide more effective inventory control and appropriate pricing to achieve better coordination for the PC product supply chain.

The subject of this paper focuses on the joint decision of dynamic inventory and dynamic pricing with demand substitution and demand forecasting. There are some studies in such fields, but there is little existing work on considering multi-product and PC product issues, which are very practical in the PC product retail context.

Table 1 Stock-Out-Based Demand Substitution Type and Structure

Substitution structure \ Related literature			Bassok et al. (1999)	Eynan and Fouque (2003)	Nagarajan (2008)
Substitution driver	*Customer-driven*				●
	Supplier-driven		●		
	Retailer-driven			●	
Demand model	Customer choice model				
	Probabilistic distribution		●	●	●
Substitution direction	One-way		●	●	●
	Two-way				
Substitution attempt	One-attempt		●	●	●
	Multi-attempt				
Number of substitutes	One product	For one product		●	●
		For multi-products	●		
	Adjacent product				
Substitution rate	Full		●		
	Partial	Constant			●
		Random		●	
		estimation			

Since McGillivray and Silver (1978) explicitly introduced the concept of substitution for the first time in an inventory problem, a large amount of literature that studies inventory decisions with stock-out-based substitution appeared. We will not review this literature in detail (we refer the reader to Nagarajan and Rajagopalan (2008) for a recent review). In Table 1, we included three papers from this literature stream to demonstrate that researchers examined this problem from different perspectives such as with a full substitution rate (see Ignall and Veinott, 1969) or a partial substitution rate (Bassok et al., 1999). Similarly, researchers considered supplier-driven substitution (Bassok et al., 1999), retailer-driven substitution (Eynan and Fouque, 2003), and customer-driven substitution (Nagarajan and Rajagopalan, 2008). The majority of the papers in this literature focus on one-way substitution scenarios for which they characterize the optimal inventory policy to provide an algorithm to compute optimal inventory levels using a single-period inventory model. Early papers, such as the one by Bassok, Anupindi, and Akella (1999), showed non-correlated demand, whereas more recent papers also studied correlated demands (Ernst and Kamrad, 2006). As far as we know, adjacent substitution has not been considered in this literature stream. We establish a multi-period model with adjacent two-way substitution.

Thus, our paper contributes to research literatures in several ways: (a) generalizes single-period inventory model with a single product or two products to a multi-period joint-optimization model with multi-product under adjacent substitution structure, meanwhile adding the service level constraint for the joint-optimization model, (b) provides an insightful interpretation of adjacent substitution under an empirical setting when few papers provide empirical information about how to measure the substitution rate in the rapidly growing literatures on inventory management, (c) and introduces, based on PC products' demand characteristics, the product life cycle (PLC) into the traditional multiplication demand model and limited order flexibility into the joint-decision model. The results indicate that this model demonstrates good feasibility and effectiveness. Moreover, simulation results demonstrate that the product life cycle significantly impacts the profit and the integrated decision model can overcome the negative impact of high demand fluctuations on profits and service levels.

PROBLEM FORMULATION AND MODEL SOLUTION

Assumptions and Notations

We consider a supply chain with two manufacturers, HP PC (personal computer) and Lenovo PC, respectively and a single retailer, a leader in China's 3C (consumer appliances, computers and communications products) chain retailers of home appliances. The retailer places orders at pre-season, and the agreement order quantity is split into multiple batches following the retailers' weekly requirements. The sum of the split orders is no more than the multiplication of ϕ and the retailer's initial agreement order, in which ϕ is an order-flexible factor set by manufacturers.

Several variants of PC basic products are considered in the joint optimization model, supplied by HP and Lenovo, respectively. We denote the set of indices of all product variants by V = {1,…,N}, and for simplicity we refer to a product variant *i* $(i = 1,2,\cdots,n)$, simply as "*product i*". Basic demand is defined as the demand of a product variant. The planning horizon for our model has *T* time periods. The basic demand D_{it} of product *i* in period $t\ (t = 1,2,\cdots,T)$ follows a random distribution with price elasticity and product life cycle factor L_{it}.

If product *i* is out of stock on a given time period *t*, we assume that a certain fraction of consumers β_{ij} will substitute from product *i* to product *j* (stock-out-based substitution). We assume that any unsatisfied demand after stock-out-based substitution would incur shortage cost. We also assume that the salvage value for each product is zero. We define *effective demand* D_{it}^{e} *of product i* in period *t* as the basic demand of a product plus any additional demand from other products due to stock-out-based substitution.

Consumers consider various attributes of a product (e.g., price, quality, and brand) when making a purchasing decision. We assume that the consumer assigns a weight $\varpi_k\ (k = 1,2,\cdots,l)$ for each attribute; and makes its final purchasing decision based on the weighted average value of these attributes, which we denote $a_i = \sum_{k=1}^{l} a_i^k \cdot \varpi^k$. Without loss of generality, we rank products in increasing order according to their average attributes value a_i. Since consumers may not be willing to substitute a product with one that has a much higher or lower average value, we assume that a consumer is willing to substitute a product with only its *adjacent products* (i.e., products ranked immediately lower or higher; note that for the products with the highest and lowest average value, there is only one choice for a substitute product). Therefore, in case of a stock-out, the consumer will consider the immediate neighbors of the desired product and possibly substitute the product with the one with the closest attribute value.

The retailer adopts a periodic review inventory replenishment policy for each product *i* that remains constant through the planning horizon (with *T* time periods); and sets the order-up to inventory level for each product in period *t* at $S_{it} = S_i \cdot L_{it}$. She then places a replenishment order Q_{it} at the end of each period *t*. The order quantity Q_{it} equals the maximum inventory level S_{it} minus on-hand inventory level at the end of period *t*.

Let C_i be the unit cost of production to the manufacturer, P_{it} be the sale price of product i at period t, h_i be the holding cost of product i during T periods, B_i be the unit shortage cost, W_i be the wholesale price, and G be the rebate provided by manufacturers.

Estimating Stock-Out-Based Substitution Rate

Due to the technology and upgrade, the PLC (Product Life Cycle) of a PC product is slightly different from the PLC of other products. Its life cycle curve shows an symmetrical sigmoid shape. Based on PC products' demand characteristics, a product life cycle (PLC) coefficient is introduced into the traditional multiplication demand model to depict the asymmetrical sigmoid function of PC products' life cycle curves.

$$D_{it} = K_i \cdot P_{it}^{\alpha_i} \cdot L\xi_i \tag{1}$$

Where K denotes the scale constant of the product, α denotes the price elasticity (sensitivity of demand derivation corresponding to the price fluctuation), L_t denotes the PLC (Product Life Cycle) function of identical products of last season to derive the demand of current season, and ξ denotes the random disturbance $\xi \sim N(\mu, \sigma)$.

From interviews with sales managers, we find out that, for the computers with similar configurations and prices, the influence on sales increments of the price reduction of HP is much more than that of Lenovo, that is to say, $\alpha_1 \geq \alpha_2$. A demand forecasting model could use history data to estimate the coefficients in equation (1). First, select from the products of last season with similar brand and configuration as well as price. Second, analyze these products' sales data. The sales are the dynamic value of the life cycle function varying with time to forecast the demand of each period divided by the average sales revenue. Then, obtain a value of the current season. Last, estimate K, α, and the mean value and standard deviation of ξ in equation (1).

Note that the effective demand of i is its basic demand sale D_{it} plus any demand from substitution demand D_{it}^{SBS} transferred from its neighborhood products that may substitute with product *i* due to a stock-out. Since stock-out-based demand D_{it}^{SBS} cannot be larger than the leftover inventory of product *i*, we define effective demand as

$$D_{it}^{\ e} = D_{it} + D_{it}^{SBS}$$

$$= D_{it} + \min[I_{it}, \sum_{i \neq j} \beta_{ij} \cdot \max(0, D_{jt} - S_j \cdot L_{jt})],\ i = \begin{cases} j+1 & \text{for } j = 1 \\ j-1,\ j+1 & \text{for } 1 < j < n \\ j-1 & \text{for } j = n \end{cases} \quad (2)$$

From assumptions, we know that stock-out-based substitution rate β_{ij} is based on the various attributes a consumer considers when making a purchasing decision and the values she assigns to these attributes. To estimate the stock-out-based substitution, a retailer can follow *three steps*.

(1) Select the number $k(k = 1, 2, \cdots, m)$ of attributes to consider and the corresponding attributes values a_i^k (each attribute value can be easily translated to a ratio value from 0 to 1, e.g., a price of \$100 for a product can be translated to 0.5 if \$200 is the sum of all prices of all products in the assortment).

(2) Estimate the weight ϖ_k for each attribute of a product. Calculate the average attribute value a_i for product *i* as $a_i = \sum_{k=1}^{l} a_i^k \cdot \varpi_k$. Estimate the partial substitution coefficient pr$_j$. As with the partial substitution coefficient, determining the appropriate number of attributes as well as the appropriate weight for each attribute must be left up to the retailer.

(3) According to our adjacent substitution assumption, the stock-out-based substitution rate b_{ij} would be larger when the difference in the average attribute value between *i* and *j* is smaller. Thus, we define β_{ij} as follows:

$$\beta_{ij} = \begin{cases} \Pr_j, & \textit{for}\ \ j = 1, i = j+1 \\ [1 - \dfrac{|a_j - a_i|}{|a_{i+1} - a_{i-1}|}] \cdot \Pr_j, & \textit{for}\ \ 1 < j < n, i = j-1, j+1 \\ \Pr_j, & \textit{for}\ \ j = n, i = j-1 \end{cases} \quad (3)$$

where $\Pr_j$ is the partial substation coefficient, which means that the consumer loyalty probability to j is judged by the sales manager’s experience.

Joint-Decision Model of Dynamic Inventory and Pricing

We build demand forecasting and an inventory control integrated decision model upon the profit functions of the retailer and the manufacturer, respectively.

1. The profit function of retailer

The profit of the retailer is a combination of the sales profit as well as the rebate (when order quantity exceeds the agreement, the manufacturer offers the rebate to the retailer) minus the purchase cost, inventory holding cost, and shortage cost. The initial order quantity that the retailer makes before sales season equals the total quantity of forecasting of each period.

Note that the order quantity Q_{it}^{e} equals the maximum inventory level S_{it} minus the on-hand inventory level at the end of period *t*. That is

$$Q_{it}^{e}(S_i) = S_i \cdot L_{it} - I_{i(t-1)}^{e} \tag{4}$$

Further, the effective on-hand inventory level and stock-out quantities are

$$I_{it}^{e} = \max(0, S_i \cdot L_{it} - D_{it}^{e}) \tag{5}$$

$$J_{it} = \max(0, D_{it}^{e} - S_i \cdot L_{it}) \tag{6}$$

Then, the total sales revenue of product i during T periods SA_i^e, the procurement cost OC^e, the shortage cost *HB*, and the inventory holding cost H_{it}^{e} are

$$SA_i^e = P_{it} \cdot \sum_{t=1}^{T} \min[D_{it}^{e}, S_i \cdot L_{it}] \tag{7}$$

$$OC_i^e = W_i \cdot \sum_{t=1}^{T} Q_{it}^{e} \tag{8}$$

$$HB_i = \sum_{t=1}^{T} B_i \cdot J_{it} \tag{9}$$

$$HB_i^e = HB_i - B_i \cdot \sum_{i \neq j} \min[I_{jt}, \beta_{ji} \cdot (D_{it} - S_i \cdot L_{it})] \tag{10}$$

$$H_i^e = h_i \cdot \frac{1}{T} \sum_{t=1}^{T} \left(\frac{S_i \cdot L_{it} + I_{it}^{e}}{2}\right) \tag{11}$$

The manufacturer offers the retailer a rebate point G_i^e unless the retailer's order quantity exceeds the agreement order; otherwise, there is no rebate:

$$G_i^e = \begin{cases} \gamma_i \cdot OC_i^e, & \sum_{t=1}^{T} Q_{it}^e \geq Q_i, 0 \leq \gamma_i \leq 0.1. \\ 0, & \sum_{t=1}^{T} Q_{it}^e < Q_i. \end{cases} \quad (12)$$

Then, the retailer's total revenue TPR_i^e is as follows:

$$TPR_i^e = SA_i^e - OC_i^e - HB_i^e - H_i^e + G_i^e \quad (13)$$

The *expected service level* of the retailer can be defined as follows:

$$ESL_i = 1 - \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\left\{J_{it} - \sum_{i \neq j} \min[I_{jt}^e, \beta_{ji} \cdot J_{it}]\right\}}{\sum_{i=1}^{n}\sum_{t=1}^{T} D_{it}^{\ e}} \quad (14)$$

2. The sales revenue, production, and rebate cost primarily comprise the profit function of manufacturers:

$$TPM_i^e = \sum_{t=1}^{T} (W_i - C_i) Q_{it}^e - G_i^e \quad (15)$$

3. The dynamic inventory and pricing integration model

Suppose that the manufacturer is the leader and the retailer is the follower of this PC supply chain. First, we propose the manufacturer's profit optimization model. The objective function is the manufacturer's profit maximization and decision variables are wholesale price and rebate coefficient. The first constraint means that the rebate coefficient which manufacturer offers retailer is ranging from 0 to 10%; the second constraint is the range of the manufacturer's acceptable wholesale price, for which $\underline{P}$ is the lower limit of the unit retail price and C_i is the unit production cost, as follows:

$$\max_{\gamma_i, W_i} TPM_i^e$$

$$\text{s.t.} \quad 0 \leq \gamma_i \leq 0.1$$

$$C_i \leq W_i \leq \underline{P} \quad (16)$$

Model Solution: We could use a numerical method to solve the manufacturer's optimal wholesale price and rebate coefficient. Searching the rebate coefficient in the closed interval [0, 0.1], as well as W_i in the closed interval [$C_i, \underline{P}$], we could finally find out the (γ_i^*, W_i^*) that satisfies both constraints while realizing the maximization of profit.

Our objective is to maximize the retailer’s total expected profit by choosing the optimal unit retail price per period P_{it} and optimal order-up-to inventory levels S_i for each product, subject to the constraints of satisfying a minimum target service level, having a flexible order quantity, and accommodating the valuing range of decision variables. Defining $\Omega \in [0,1]^m$ as the *m*-dimensional space of [0, 1] intervals, we therefore can formulate the following nonlinear integer programming model to express the problem of jointly determining pricing and inventory replenishment:

$$\max_{P_{it},S_i} TPR_i(\beta_{ij}) = \sum_{i=1}^{n}\sum_{t=1}^{T} TPR_i(\beta_{ij})$$

$$\text{s.t.} \qquad ESL \geq TSL$$

$$\sum_{t=1}^{T} Q_{it}^{e}(S_i) \geq \phi_i \cdot Q_i$$

$$S_i > 0$$

$$W_i \leq P_{it} \leq \bar{P}$$

$$0 \leq \beta_{ij} < 1 \text{，} \beta_{ij} \text{ derived from (3)}$$

$$\sum_{k=1}^{l} W_k = 1 \text{，} (\varpi_1, \varpi_2, ..., \varpi_m) \in \Omega \tag{17}$$

Model Solution: We also could use the numerical method to solve the retailer’s optimal inventory level and retail price. Searching P_{it} in the closed interval [$W_i, \bar{P}$], and S_i in the closed interval (0, Q_i], we finally find out the (S_i^*, P_{it}^*) that satisfies all constraints while realizing the maximization of the retailer’s profit.

Solution for this Stackelberg Game Model:

Step 1: Based on (16), the manufacturer determines an initial rebate coefficient and a wholesale price (γ_i^1, W_i^1) which the superscript represents the optimum decision value to the first calculation.

Step 2: After learning the rebate coefficient and wholesale price, based on (17), the retailer could find the optimum (S_i^1, P_{it}^1).

Step 3: After acquiring the latest replenishment and sales price, based on (16) again, the manufacturer could find out the optimum (γ_i^2, W_i^2).

Step 4: Return to Step 2.

The computation stops until the sequential calculation reaches a balanced error no less than a given constant ε. Then, we obtain the optimal solution ($\gamma_i^*, W_i^*, S_i^*, P_{it}^*$). Finally, we could obtain the maximal profit of the members of the supply chain as well as the entire supply chain.

APPILICATION STUDY OF MODEL

Parameter Estimation

We take logs of both sides of equation (1) in order to use Eviews to estimate the coefficients by the regression analysis method. The demand function is transferred to a multi-element linear model as follows:

$$Ln(D_{it}) = Ln(K_i) - \alpha_i Ln(P_{it}) + Ln(L_{it}) + Ln(\xi_i) \qquad (18)$$

The data of this model are sourced from a chain retailer of home appliances in Shanghai. Our test sample consists of PC sales data during one year and three months (62 weeks in all). We chose products from HP and Lenovo. Product 1 and product 3 belong to the HP brand; product 2 and product 4 belong to the Lenovo brand. Based on the basic sales data of all products, see Table 3, we use Eviews's multi-element linear regression to estimate the demand model parameters and other parameters shown in Table 2.

Table 2 Value of Parameters in the Model

	Parameter	**Product 1**	**Product 2**	**Product 3**	**Product 4**
Demand	K_i	16366.63	595.8566	20889.46	483.9589
	α_i	0.79	0.50	0.80	0.42
Cost	W_i	0.80*$\overline{P_1}$	0.85*$\overline{P_2}$	0.80*$\overline{P_1}$	0.85*$\overline{P_2}$
	C_i	0.80*W_1	0.85*W_2	0.80*W_1	0.85*W_2
	h_i	0.20*$\overline{P_1}$	0.15*$\overline{P_2}$	0.20*$\overline{P_1}$	0.15*$\overline{P_2}$
Constraint	B_i	10000	10000	10000	10000
	TSL_i	0.90	0.85	0.90	0.85
Sub-rate	$\Pr_i$	0.782	0.875	0.782	0.875

Table 2 Value of Parameters in the Model (Continued)

	Parameter		Product 1	Product 2	Product 3	Product 4
Attribute	Brand (ϖ_1 =0.3)		HP	Lenovo	HP	Lenovo
			0.218	0.125	0.218	0.125
	Property (ϖ_2 = 0.4)	CPU 0.4	AMD Sempron 3200+ 1.8 GHz	Intel Pentium4 516 2.8 Ghz	AMD Sempron 3200+ 1.8 GHz	AMD Athlon64 3000+ 1.8Ghz
			0.7	0.5	0.7	0.6
		Memory 0.2	512M	256M	256M	256M
			0.8	0.4	0.4	0.4
		VGE Card 0.2	Onboard VGA Controller	nVIDIA GeForce 6200TC 128M	Onboard VGA Controller	nVIDIA GeForce 6200TC 128M
			0.3	0.6	0.3	0.6
		HD 0.2	80g/7200	80g/7200	80g/7200	80g/7200
	Price (ϖ_3 =0.3)		$1-[P_1/\sum_{i=1}^{4}P_i]$	$1-[P_2/\sum_{i=1}^{4}P_i]$	$1-[P_3/\sum_{i=1}^{4}P_i]$	$1-[P_4/\sum_{i=1}^{4}P_i]$
Average Attributes Value			0.525	0.472	0.491	0.482
Rank			1	4	2	3

We obtain the products' ranks by the attribute value and corresponding weight. From the brand loyalty, which referred to ZOC's survey data of 2005, we find out the substitution rate of the PC product. ZOC divided the attributes into brand, property, and price, and the property is further segmented into CPU, memory, VGE card, and HD.

Table 3 Weekly Sales Data from September to December

The Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
D_1	2	3	6	12	25	40	50	60	62	50	35	24	12	10	5	3	3
D_2	10	12	14	19	26	30	40	46	55	35	20	10	6	4	2	2	2
D_3	3	6	7	8	14	17	24	27	25	35	40	23	12	8	5	2	2
D_4	2	2	3	5	9	14	18	22	26	35	20	15	10	6	4	3	1

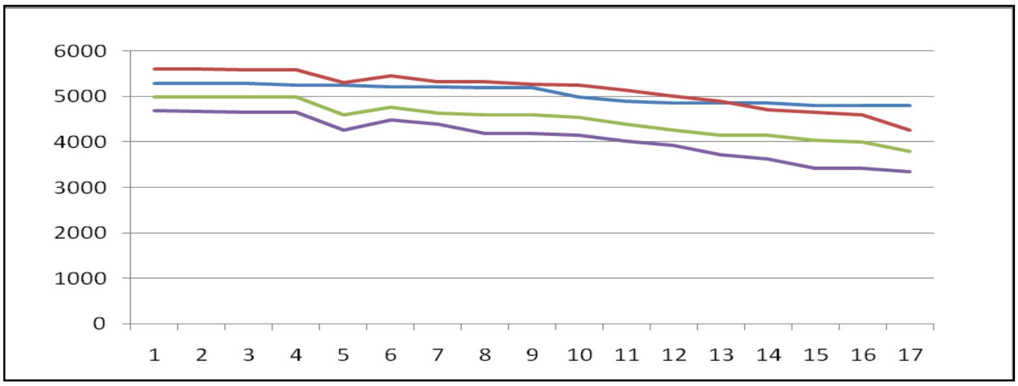

Figure 1 Price Fluctuation Curves of Four Products of HP and Lenovo

Figure 2 and Figure 3 show that the fitted demand curve tracks demand better; and the goodness-of-fit of the four products reaches $RS_1 = 0.8947$, $RS_2 = 0.9593$, $RS_3 = 0.8998$, and $RS_4 = 0.7136$, respectively.

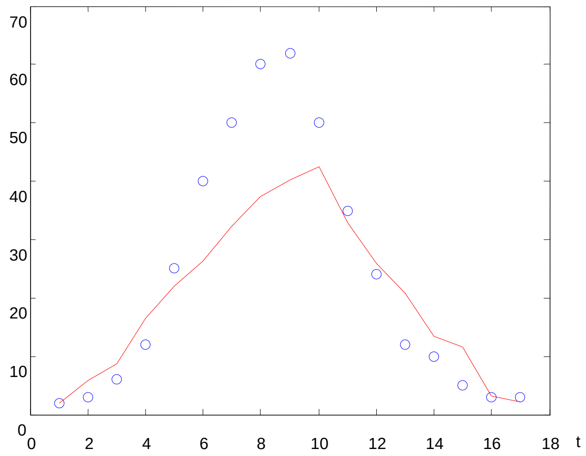

Figure 2 Demand Fitting Curve of Pro-1

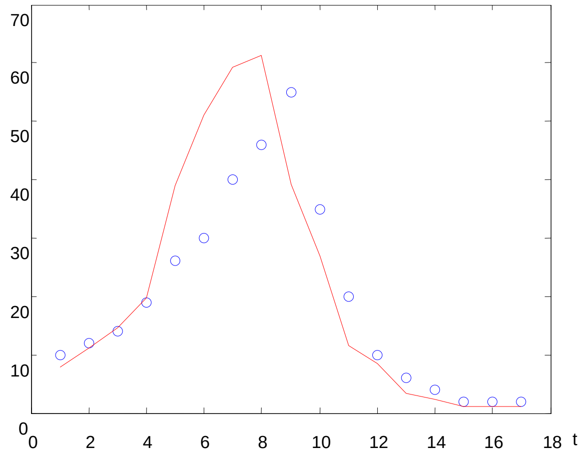

Figure 3 Demand Fitting Curve of Pro-2

Note: The circles represent the original demand data, and the straight line represents the fitted demand curve.

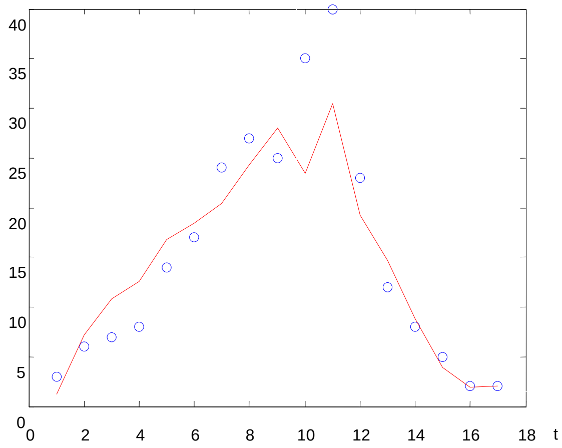

Figure 4 Demand Fitting Curve of Pro-3

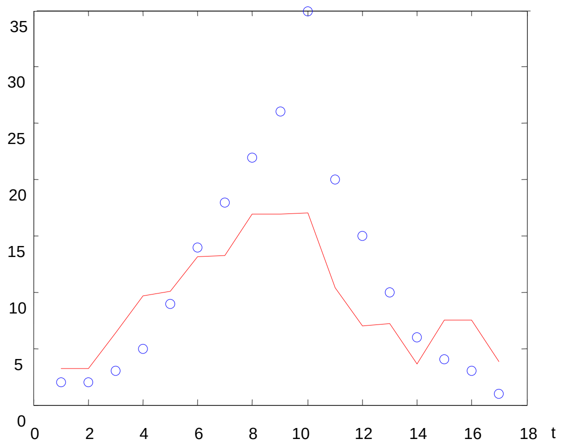

Figure 5 Demand Fitting Curve of Pro-4

Application of Model

Figures 6-9 show that the trends of markdown of both HP and Lenovo are unanimous. Under ORI mode, the sales price is identical with the price under ORI2 mode; therefore, the ORI line and ORI2 line are overlapped in figures 6-9. Under STA2 strategy, the price of product 1 is apparently higher than the other three products'. Because there is substitution between product 1 and product 2 that improves the combination of these two products, when both inventory level and sales of product 1 happen to reduce, the supply chain of HP always performs better. Product 2 meets the majority part of the shortage of product 1; in consequence, the order-up-to-inventory level of product 2 surges under STA2 strategy. Meanwhile, the increase of sales leads to the reduction of price. There is no significant change in the maximum inventory level and sales of product 3 so that price changing under STA and STA2 strategies is similar.

Though there is also no significant change of price, the demand of product 4 reached peak during the 10th week, leading to the demand's exceeding the supply under STA strategy. That is to say, the shortage of the product produced a small price ascent during the 10th week.

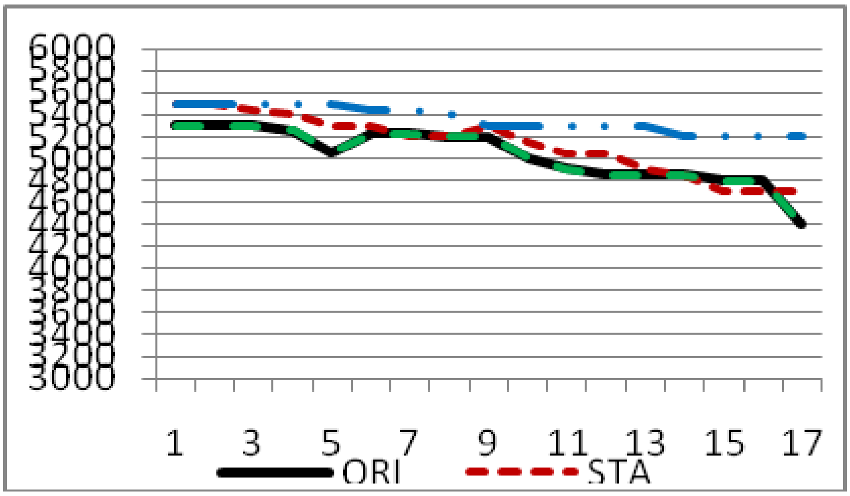

Figure 6 Sales Price Fluctuation of Pro-1

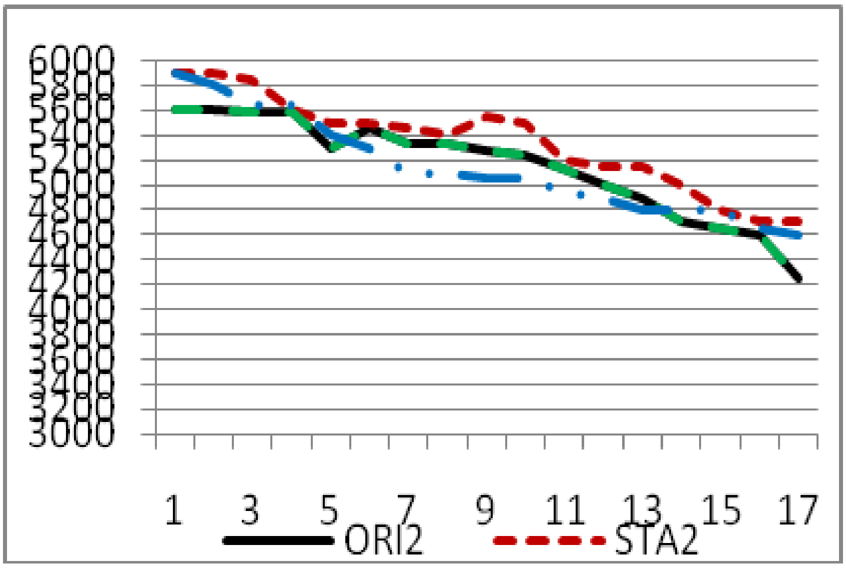

Figure 7 Sales Price Fluctuation of Pro-3

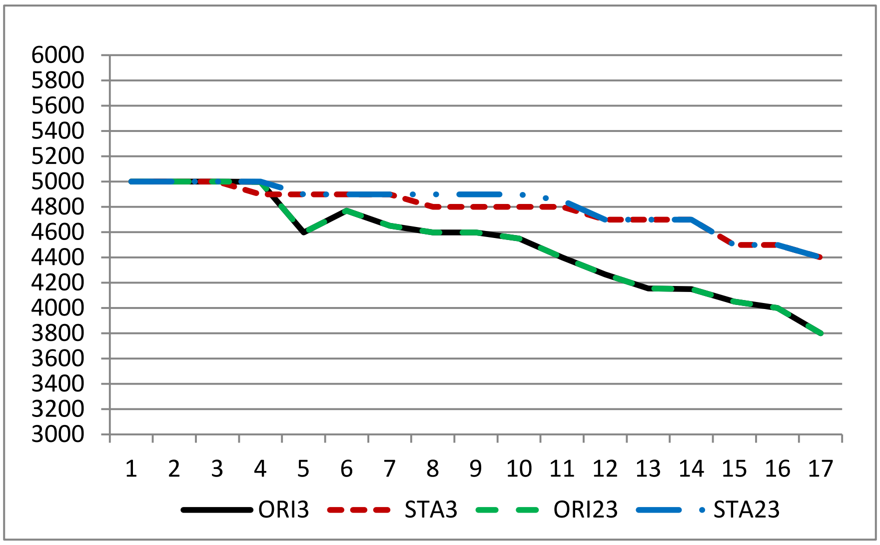

Figure 8 Sales Price Fluctuation of Pro-2

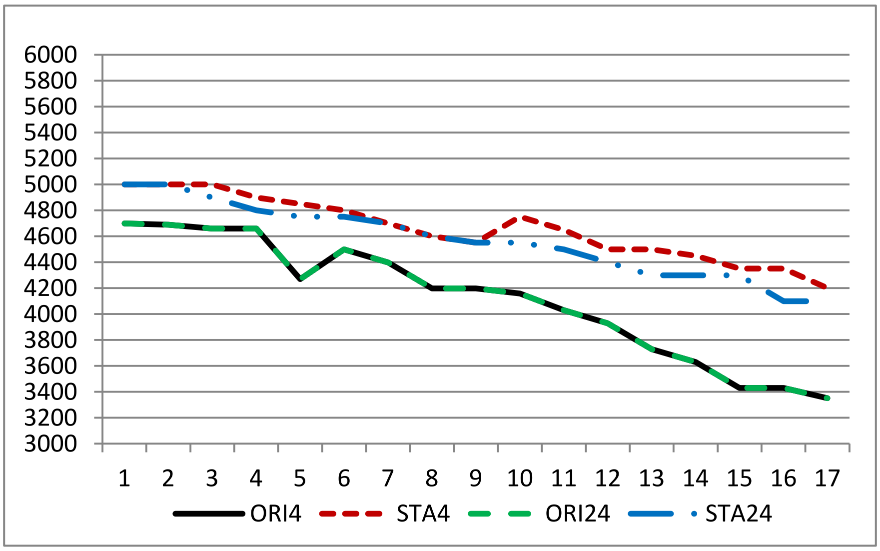

Figure 9 Sales Price Fluctuation of Pro-4

- Supply Chain Profit

On one hand, the master game STA2 strategy considering substitution acquires a better profit in terms of the whole profit of the supply chain because of the quick replenishment of other products to satisfy the shortage, which lowers the shortage cost; on the other hand, under our numerical method, the final result realizes the maximal profit of the whole supply chain; HP and Lenovo have a 2.25% and 0.38% improvement of supply chain profit, respectively.

- Service Level

The average supply chain service level of HP increases by 3.67% from ORI to ORI2 and 2.52% from STA to STA2. The average inventory level of HP stays at 21 because that supply chain is in a balanced position; manufacturers and retailers reach a balance between supply and demand. The average supply chain service level of Lenovo also increases. The average inventory level of Lenovo sees a higher fluctuation under STA because of the shortage under ORI—while the increase of maximum inventory level makes up the shortage. Furthermore, considering the substitution, ORI2 and STA2 could achieve a higher profit under a lower average inventory level.

Simulation Analysis

- Product Life Cycle Sensitivity Analysis

The PLC of PC products exerts a significant impact on demand. We will analyze the sensitivity of PLC. Figures 10–11 show that B1 represents the original PLC function of the model, which has a similar brand and configuration and could effectively forecast the demand of product 1. The data of R1 and R2 also come from the last sales season, but the attributes are very different from product 1’s.

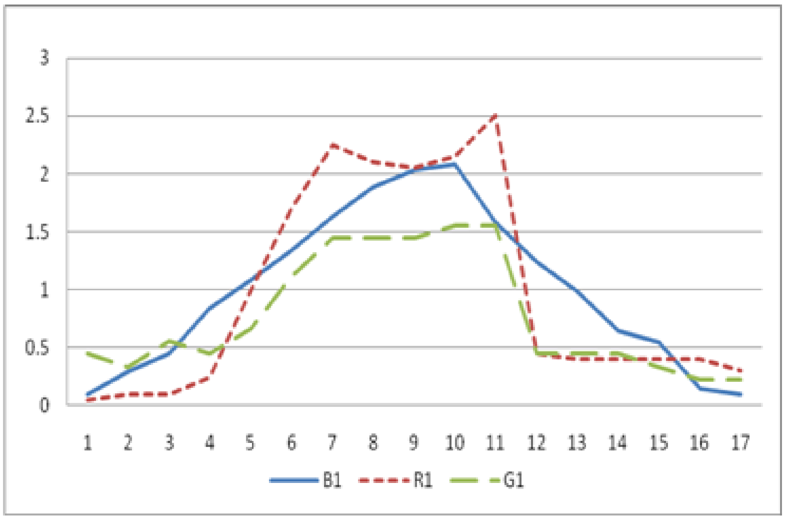

Figure 10 PLC Curves of 3 Products, HP

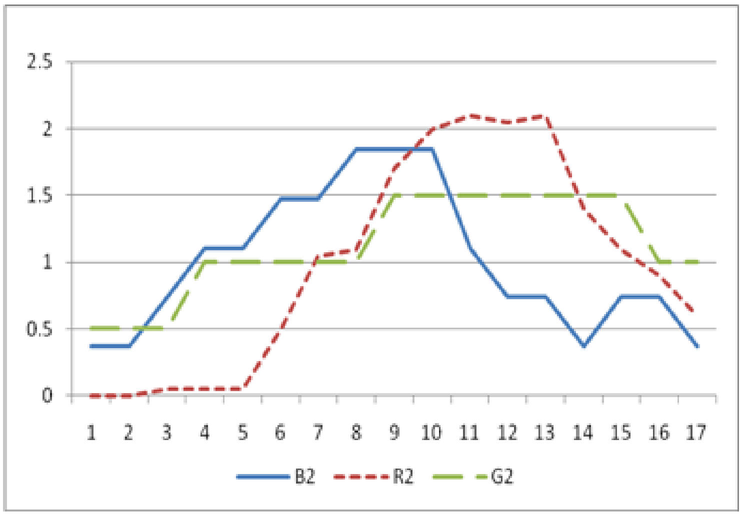

Figure 11 PLC Curves of 3 Products, Lenovo

Table 4 makes a comparison of ORI and STA. From the vertical review, whichever strategies were employed, we could achieve a better profit than in R1 and G1 if we choose B1 as the PLC function. Furthermore, we find that PLC has appreciable impact on demand, profit, and service level, so if we use a suitable PLC function, we could achieve a better demand forecast. Therefore, only through the reasonable judgment of the market manager on product familiarity and experience can we choose a better PLC function or else a worse profit. From horizontal review, for the original strategy and Stackelberg game, no matter how the PLC function changes, table 4 shows that the Stackelberg game is always achieving a better profit and the maximum goodness-of-fit of 34.46%. Therefore, the Stackelberg game model is doing well.

Table 4 Sensitivity Analysis of PCL L_t on Demand and Profit

		D_{it}		TPR_i		
L_{it}	**Product**	**ORI**	**STA**	**ORI**	**STA**	**(2) -(1) /(2)**
B1	**Pro. 1**	353	309	**3.534e+005**	**4.608e+005**	13.13%
R1		377	297	3.4525e+005	4.4043e+005	21.61%
G1		275	238	2.6080e+005	3.1461e+005	17.10%
B2	**Pro. 2**	188	171	**1.0567e+005**	1.6123e+005	**34.46%**
R2		193	159	7.0538e+004	1.5660e+005	54.95%
G2		218	191	9.1894e+004	1.9960e+005	**53.96%**

- Partial Substation Coefficient Sensitivity Analyses

The substitution of products occurs when a certain product is out of stock. Both the substitution quantity and substitution pattern are highly related with the substitution rate $\Pr$. We will study the effect of a partial substitution rate on profit and service level. $\Pr$ possesses two characteristics: (1) $\Pr$ is connected with brand loyalty, and the higher the brand loyalty, the smaller the $\Pr$. (2) $\Pr$ and attributes value decide β_{ij} together, and $\Pr$ reacts upon product demand through β_{ij}, then reacts upon profit.

We analyze the sensitivity of $\Pr$ in the following 4 conditions:
(1) HP's and Lenovo's Prs are increasing in equal fluctuation ranges at the same time, (2) HP's and Lenovo's $\Pr$ are decreasing in equal fluctuation range at the same time, (3) HP's $\Pr$ is increasing while Lenovo's $\Pr$ is decreasing, (4) HP's $\Pr$ is decreasing while Lenovo's $\Pr$ is increasing.

Table 5 shows that when HP's and Lenovo's Prs are increasing or decreasing at the same time, HP's and Lenovo's profits of the whole supply chain are correspondingly increasing or decreasing. This is because the change of $\Pr$ leads to the change of β_{ij} which, regardless of increase or decrease, will directly influence the change of demand in an identical trend. When HP's $\Pr$ changes in an opposite trend with Lenovo's $\Pr$, one side will achieve a lower profit (decreasing $\Pr$). This is because if HP's $\Pr$ is increasing, it leads to the increment of substitution demand as well as profit. However, the decreasing of Lenovo's $\Pr$ introduces more substitution coming from HP. If Lenovo appears to be out of stock without effective replenishment, there will be more profit for HP and less for Lenovo.

CONCLUSIONS

This paper established a Stackelberg game model for the PC industry with two manufacturers and one retailer. Under the assumption that the two manufacturers offer a series of products with substitution and identical configuration, Product Life Cycle (PLC) is effectively introduced into the traditional multiplication demand model, and a multi-period dynamic inventory and pricing integrated decision model of a PC product is established based on demand forecasting. At the same time, a retailer would determine the optimal inventory level and the optimal sales price during each period according to the model, and the manufacturers decide the wholesale price and rebate depending upon the total amount of the retailer's order. We apply this model to a large home appliance retailer. We found that this model is entitled with better feasibility and application effect. Under STA2 strategy, both the whole supply chain profit and service level are the most optimal strategy, which means that it is significant and essential to incorporate the substitution of demand into inventory and pricing decision. We also find that the estimation of PLC and brand loyalty influence the profit and service level greatly. The future study could extend to incorporate PLC learning, brand loyalty, and other marketing problems into supply chain management research.

Table 5 Sensitivity Analysis of Partial Substitution Coefficient on Supply Chain Profit

Range	Partial Substitution Coefficient	Brand	Profit (unit: RMB)			
			ORI	ORI2	STA	STA2
No fluctuation	0.782	HP	1,286,400	1,436,200	1,707,100	1,746,400
	0.875	Lenovo	556,700	587,250	899,800	903,200
Increase by 10% both	0.8602	HP	1,298,700	1,435,300	1,709,900	1,751,700
	0.9625	Lenovo	566,700	595,300	910,800	911,100
Decrease by 10% both	0.7038	HP	1,186,400	1,339,200	1,615,200	1,633,700
	0.7875	Lenovo	469,800	501,100	800,400	803,200
HP increase by 10%, Lenovo decrease by 10%	0.8602	HP	1,290,200	1,446,800	1,711,500	1,750,000
	0.7875	Lenovo	452,300	500,200	799,900	768,500
HP decrease by 10%, Lenovo increase by 10%	0.7038	HP	1,154,300	1,237,200	1,595,200	1,603,700
	0.9625	Lenovo	586,700	621,500	933,800	953,100

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